Spin-Crossover from a Well-Behaved, Low-Cost meta-GGA Density Functional

Daniel Mejía-Rodríguez* and S. B. Trickey*

ABSTRACT: The recent major modification, r^2SCAN, of the SCAN (strongly constrained and appropriately normed) meta-GGA exchange-correlation-functional is shown to give substantially better spin-crossover electronic energies (high spin minus low spin) on a benchmark data set than the original SCAN as well as on some Fe complexes. The deorbitalized counterpart r^2SCAN-L is almost as good as SCAN and much faster in periodically bounded systems. A combination strategy for the balanced treatment of molecular and periodic spin-crossover therefore is recommended.

INTRODUCTION

Context. The essential physical trait of a spin-crossover (SCO) molecule is a small energy difference between the ground state of one spin and an excited state of a different spin. Small in this context typically means a few kcal/mol (i.e., a few hundred meV). Calculation of such differences is challenging. An added challenge is that spin-crossover is of greatest interest in condensed phases. The interest arises both because of the intrinsic complexity of bistability and because of its importance for functional materials in technological applications such as switchable-spin memories. For an early treatment see ref 1, and for a very recent one see ref 2. Predictive calculation protocols therefore must be equally accurate for both isolated molecules and their condensed phases.

Meeting that challenge has been difficult. It is not our purpose to survey the literature. For that, see refs 3–11. The last-mentioned of these is particularly relevant. It presented a database of 20 molecules in which SCO arises from a first-row transition metal. Against that database, the authors of ref 11 tested several rather sophisticated density functional approximations (DFAs) for exchange and correlation (XC) and concluded that the hybrid Tao–Perdew–Staroverov–Scuseria (TPSSh)12,13 DFA was best overall.

The focus on DFAs stems from the need for affordable calculations both on large molecules and on their condensed aggregates. Refined wave function methods are applicable, though costly, in SCO molecules. They are prohibitively slow in the condensed phases. In principle, density functional theory (DFT) methods should be applicable to both. Until recently, however, all affordable, “lower-rung”15,16 DFAs have exhibited bias to either the molecular or the condensed side.

The recommendation of TPSSh is itself somewhat problematic. The drawback that is relevant here is its hybrid character, namely, inclusion of 10% single-determinant exchange (often inaccurately called Hartree–Fock or exact exchange; both terms have precise, well-defined meanings that are not met by a hybrid DFA).

The nonhybrid antecedent of TPSSh, TPSS, is a meta- Generalized Gradient Approximation (meta-GGA). In meta-GGAs, chemically distinct electron density inhomogeneities are recognized by the use of so-called indicator functions. In the case of TPSS there are two. Based on their values, the meta-GGA switches between a nonempirical GGA DFA that is constructed to work well with molecular-like environments and another for condensed-phase environments.

Largely for reasons of accuracy, TPSS has been supplanted by a more refined meta-GGA called SCAN, for “strongly constrained and appropriately normed”15,16. It uses only one indicator function, denoted α(r). With comparatively few exceptions (e.g., ref 17), SCAN has proven successful in predicting a wide variety of molecular and condensed-phase properties. That success is a consequence of the physical realism associated with the enforcement in SCAN of all the rigorous constraints that a meta-GGA can meet, along with calibration to the energies of selected primitive physical systems (the “appropriate norms”; see the Supporting Information to ref 15).

SCAN and Spin-Crossover. Motivated by other successful uses of SCAN, Cirera and Ruiz18 tested it recently against the 20-molecule SCO database of ref 11. Their conclusion was that SCAN “...gives the right ground state for the whole set of test cases” and is “...the unique pure DFT functional to provide with comparable results for such a challenging test.” All of the systems...
have low-spin (LS) as the ground state, as indeed is found by SCAN. However, the SCO energy differences,

$$\Delta E_{HL} := E_{HL} - E_L$$

(1)

with $E_{HL}$ ($E_L$) the high-spin (low-spin) total energy, from SCAN are only semiquantitative at best. In some cases they are off by as much as a factor of 2 or more. Note that these comparisons were with respect to TPSSH results for $\Delta E_{HL}$. Those values themselves lead to an overestimation of the crossover temperature. A technical difficulty is that SCAN calculations required dense radial integration grids. An uncomfortable aspect is that the best range of $\Delta E_{HL}$ values from SCAN were generated with a suboptimal (not fully converged) grid.

Grid density and SCF convergence difficulties with SCAN already had become well-known among practitioners. Those problems were addressed by Bartok and Yates with regularized SCAN (rSCAN). It refined $\alpha$ and smoothed the SCAN switching function to yield improved computational behavior. Though rSCAN preserves the good molecular bond lengths and vibrational frequencies given by SCAN, it sacrifices the SCAN performance for benchmark molecular heats of formation. In periodic solids, SCAN and rSCAN are about the same for lattice constants and cohesive energies, on a 55 solid test set and for bulk moduli on a 44 solid set.22

Very recently Furness et al.23 have cured the deficiencies of rSCAN by constructing a similar regularization that restores all but one of the constraints satisfied by SCAN but violated by rSCAN. The regularized-restored SCAN functional (r2SCAN) that results combines the strong performance trends of SCAN relative to molecular and solid data sets with the numerical stability of rSCAN.

A separate conceptual and computational issue of meta-GGAs in general is their explicit dependence upon the Kohn–Sham (KS) orbitals. As a matter of practice, the computational costs from that dependence lead to the use of the generalized KS (gKS) equations rather than the multiplicative potential of the ordinary KS equation. There is both a difference of content and a computational cost penalty for gKS compared to KS. We had addressed both those issues by deorbitalization, that is, the replacement of the orbital dependence with a function of the density, its gradient, and its Laplacian. That gave the SCAN-L DFA. Except for elemental 3d magnetic solids, SCAN-L delivered essentially the same performance as SCAN. It should be faster than SCAN, but in practice numerical instabilities caused very slow SCF convergence. Very recently we found that the greatly improved numerical stability of r2SCAN is preserved under deorbitalization to yield r2SCAN-L. In solid calculations, it runs almost four times faster than rSCAN.28

The advent of r3SCAN and r3SCAN-L makes it imperative to investigate their SCO performance on the ref 11 data set to see if the changes from SCAN and SCAN-L (respectively) affect the delicate energy differences involved.

We note that validation and testing of any DFA with regard to SCO must be preceded by standard screening. Any plausible DFA candidate for SCO first must have given acceptable accuracy for molecular dissociation energies, bond lengths, and fundamental vibrational frequencies and must give acceptable crystal structures, cohesive energies, and bulk moduli, all against widely used databases. If the fundamental molecular vibration frequencies and bulk moduli are of decent quality, it is plausible that the enthalpic contributions to the SCO temperature $T_{1/2}$ will be reasonable. Notice that the essential untested ingredient from routine DFA screening is $\Delta E_{HL}$. If the candidate DFA delivers bad values for that difference, the only way it could deliver good $T_{1/2}$ values would be by compensating error, i.e., right answers for wrong reasons. We focus therefore on $\Delta E_{HL}$. For the sake of delineating DFA performance in difficult spin systems, we also study the Cr d dissociation curve.

### COMPUTATIONAL METHODS

Molecular calculations were done with a locally modified developers’ version of the NWChem code using the unrestricted KS procedure, of the def2-TZVP basis set in spherical representation, and the FINE numerical integration grid. The number of radial shells and the corresponding Lebedev angular points31–36 per radial shell vary depending on the atom type, as shown in Table 1. Previously we have shown that this

<table>
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<th>angular points</th>
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<tr>
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<td>60</td>
<td>590</td>
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<tr>
<td>B, C, N, O</td>
<td>70</td>
<td>590</td>
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<td>P, S</td>
<td>123</td>
<td>770</td>
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<td>Cr, Mn, Fe, Co, Br</td>
<td>140</td>
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<td>I</td>
<td>141</td>
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grid density is good enough to integrate both r2SCAN and r2SCAN-L XC potentials and energies. SCAN calculations used a custom-defined grid with 200 radial shells and 590 angular points per shell.

Moreover, all calculations used Weigend’s Coulomb-fitting basis set for the density fitting scheme. Default options for guess density, convergence stabilization and acceleration techniques, and convergence criteria for both electronic and ionic relaxations were used.

The D3(BJ) empirical dispersion correction, with parameters optimized for SCAN,44 was tried as an exploratory step. We remark that both r2SCAN and r2SCAN-L should include some midrange dispersion by construction, so the D3(BJ) corrections are rather small.

Nine SCO systems in the Cirera–Via-Nadal–Ruiz database11 database are positively charged (2 MnIII, 3 FeIII, 1 FeII, and 3 CoII). None of the counterions were included in the calculations. That choice is consistent with the original database and, as well, avoids the intrinsic bias toward noncharged species in the gas phase. The omission corresponds to removal of 10% or less of the total atomic count for most of the charged complexes. However, the 45 atoms of the tetraphenyl borate anion originally present in the FeIII system labeled S9 account for almost 40% of the total number of atoms of that system.

The spin-state energetics for the four Fe complexes, three FeIII and one FeI, recently benchmarked by Rad63 also were computed. Geometries of all four were reoptimized under the same symmetry constraints as in ref 43 using the same settings as for systems in the ref 11 database.

The chromium dimer potential energy curves were obtained using a modified version of VASP 5.4.4 using the 14-electron projector augmented-wave (PAW) data set. The dimer was aligned along the z-axis inside a large 12 × 12 × 15 Å box. The calculations used the accurate precision setting and a 600 eV kinetic energy cutoff and included aspherical corrections inside the PAW spheres.
RESULTS AND DISCUSSION

Table 2 and Figure 1 show $\Delta E_{\text{HL}}$ in kcal/mol, obtained with SCAN, rSCAN, r$^2$SCAN, and r$^2$SCAN-L. Results from ref 18 for TPSSh and SCAN are included for comparison. SCAN results correspond to the denser numerical integration grid (SG-2) which is made up of 75 radial shells with 302 Lebedev angular points per shell. Our $\Delta E_{\text{HL}}$ values obtained with SCAN are around 1.5 kcal/mol from those of ref 18. This difference was expected based on our previous studies (see refs 20 and 28).

Reassuringly, r$^2$SCAN gives the correct low-spin configuration for the ground state for all 20 complexes, with all $\Delta E_{\text{HL}}$ values inside a 10 kcal/mol energy window proposed by Cirera and Ruiz (compare Figure 1 with Figure 3 of ref 18). Also striking is the fact that the r$^2$SCAN DFA yields a marked reduction of the predicted $\Delta E_{\text{HL}}$ compared to SCAN. The r$^2$SCAN values are, in fact, slightly smaller than those predicted by the DFA hybrid TPSSh.

In contrast, the deorbitalized version, r$^2$SCAN-L, gives $\Delta E_{\text{HL}}$ larger than, but still comparable to, the values obtained with SCAN. The advantage of r$^2$SCAN-L is mainly the potential speedup one can achieve by means of its local multiplicative potential. We return to this point below.

Table 2 also shows that the inclusion of empirical dispersion corrections via the DFT-D3 approach$^{40,41}$ changes $\Delta E_{\text{HL}}$ values by about 0.5 kcal/mol, with larger effects for the Co systems. We stress that forces from the r$^2$SCAN+D3 combination were included during geometry optimizations. We did not find unrealistic geometries such as reported in ref 11. Larger effects generally are seen when the DFT-D3 correction is used only for single-point energies at the corresponding uncorrected DFA minima (see for example ref 18).

In order to obtain further insight about the specific changes that lead to the drastic performance differences among these closely related DFAs, we also tried rSCAN calculations. As Figure 1 and Table 2 show, rSCAN does almost as well as r$^2$SCAN for the majority of systems but gives the wrong sign for six of them. The most notable failures occur in FeII d$^6$ systems (S11–S15). In them, rSCAN overstabilizes the high-spin state by as much as 54 kcal/mol.

It is interesting to note that, barring rSCAN results for Fe(II) systems, the S9 $\Delta E_{\text{HL}}$ is the largest of the set for the TPSSh, r$^2$SCAN, r$^2$SCAN-L, and rSCAN DFAs. This may be a direct consequence of the effects that the missing counterion can have on the overall structure and energetics of the complex (see Computational Methods), but investigation of the issue is outside the scope of this work.

Table 2. High-Spin to Low-Spin $\Delta E_{\text{HL}}$ Energies (kcal/mol)$^a$

<table>
<thead>
<tr>
<th>System</th>
<th>TPSSh$^b$</th>
<th>SCAN$^b$</th>
<th>SCAN$^c$</th>
<th>rSCAN</th>
<th>r$^2$SCAN</th>
<th>r$^2$SCAN+D3</th>
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<tr>
<td>S1</td>
<td>6.54</td>
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<td>6.41</td>
<td>6.90</td>
<td>13.62</td>
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$^a$Systems are labeled as in Cirera and Ruiz.$^{18}$ $^b$From ref 18. $^c$This work.

Figure 1. Electronic low-spin–high-spin energy differences $\Delta E_{\text{HL}}$ in kcal/mol. The colored dots correspond to the actual individual results, while the large black dot corresponds to the mean $\Delta E_{\text{HL}}$ obtained with each functional. The red box is the same as used in ref 18 to indicate the region where the electronic energy difference can be compensated by the entropy in usual SCO systems. Note that the rSCAN violin was cut in order to enhance the visibility of other results. TPSSh and SCAN$^a$ are from ref 18. All other are this work.
For thoroughness, we augmented the 20-system data set with the four systems proposed by Radon in ref 43. Radon gave $\Delta E_{\text{HL}}$ values by removing environmental effects from experimental data in order to have a straightforward reference for comparison. Table 3 shows $\Delta E_{\text{HL}}$ values for those systems (labeled as S21–S24) and the corresponding reference reported as extrapolated from experiment. Note that system S24 is the same as S8 reported in Table 2. The TPSSh and TPSSh+D3 values taken from ref 43 are from nonrelativistic calculations without corrections for spin contamination. Also note that $\Delta E_{\text{HL}}$ for S21 and S22 corresponds to vertical excitation energies, so the DFT-D3 corrected and uncorrected values are exactly the same. Again, we see that $r^2$SCAN gives the correct sign and the DFT-D3 corrected and uncorrected values are exactly the same.

Based on the foregoing results, we can summarize our findings as follows:

- The reduced $\Delta E_{\text{HL}}$ values (compared to those from SCAN) obtained with rSCAN and r$^2$SCAN mean that the seemingly small changes made in the SCAN switching function are responsible for the majority of the errors.
- The failures obtained from rSCAN but not with r$^2$SCAN highlight the importance that constraint satisfaction has in ensuring the maximum scope of validity for a given DFA.
- The larger $\Delta E_{\text{HL}}$ values obtained with SCAN-L and r$^2$SCAN-L also illuminate the importance of the switching function in the prediction of spin-state energetics. Although the deorbitalization procedure does not change the switching function directly, the differences between the approximated iso-orbital indicator $\alpha_z$ and the original one modify, indirectly, its behavior.

**CONCLUSIONS AND OUTLOOK**

We have shown that the $r^2$SCAN DFA provides a quantitatively correct ground state for all molecules in the SCO database put forth in ref 11 as well as on the four Fe complexes in ref 43. Furthermore, $r^2$SCAN apparently is the only comparatively simple DFA, including hybrid ones, that gives all high-spin to low-spin energy differences $\Delta_{\text{HL}}$ inside what is believed to be the appropriate energy range. On the basis of that accuracy and comparatively modest computational costs, we therefore recommend, strongly, the use of $r^2$SCAN to describe 3d SCO systems.

Though the accuracy for $\Delta E_{\text{HL}}$ provided by the deorbitalized version, $r^2$SCAN-L, is not as good as what r$^2$SCAN gives, it is useful that $r^2$SCAN-L does perform on par with the accuracy from the original SCAN but without the numerical integration issues. The advantage of $r^2$SCAN-L is its substantially lower computational costs in codes that use fast-Fourier transforms to obtain the appropriate derivatives of the density. In those codes, the local multiplicative potential of $r^2$SCAN-L can achieve calculations as much as four times faster than with r$^2$SCAN. That provides a major opportunity. The key to it is that the sacrifice in bond length and vibrational frequency accuracy in going from r$^2$SCAN to r$^2$SCAN-L is much smaller than the $\Delta E_{\text{HL}}$ accuracy sacrifice. The strategy we recommend therefore is to do geometry optimizations (either molecular or solid) with r$^2$SCAN-L and then do a single-point calculation with r$^2$SCAN to determine $\Delta E_{\text{HL}}$. We have that strategy under investigation but point out that an analogous approach, using SCAN and SCAN-L, has proven to be successful.

While r$^2$SCAN is much better for SCO on the Cirera–Via–Nadal–Ruiz and Radon data sets and therefore we recommend it, we do so with caution. r$^2$SCAN is not perfect for magnetization nor is r$^2$SCAN-L. See Figure 2 for a comparison of r$^2$SCAN and r$^2$SCAN-L results with those from other DFAs for the famously difficult case of Cr2 dissociation, an issue recently readdressed in ref 51. (Note that the corresponding figure in ref 51 displays the experimental data incorrectly by a factor of 2.) The reasons for the superior performance of the GGA functional PBE compared to any of the meta-GGA functionals on this system remain obscure to us and to other DFA developers (John Perdew, private communication). Another aspect of the obscurity is that, distinct from SCAN, r$^2$SCAN and both deorbitalized functionals do not magnetize benzene or graphene. This slight improvement is also seen in the solid phase, since both r$^2$SCAN and r$^2$SCAN-L improve upon SCAN (see Table 3 of ref 28). Caution is still warranted in the use of r$^2$SCAN and r$^2$SCAN-L according to the protocol we have proposed here.

**ASSOCIATED CONTENT**

 Supporting Information The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acs.jpca.0c08883.

- r$^2$SCAN optimized structures for the 23 complexes studied (ZIP)
- Chemical formulas for the 23 complexes studied (PDF)

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Notes
The authors declare no competing financial interest.

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